xtserialpm: A portmanteau test for serial correlation in a linear panel model

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Abstract. We introduce the command xtserialpm to perform the portmanteau test developed in Jochmans (2019). The procedure tests for serial correlation of arbitrary form in the errors of a linear panel model after estimation of the regression coefficients by the within-group estimator. The test is designed for short panels and can deal with general missing-data patterns. The test is different from the related portmanteau test of Inoue and Solon (2006) that is performed by xtistest (Wursten 2018) in that it allows for heteroskedasticity. In simulations documented below, xtserialpm is found to provide a much more powerful test than xthrtest (Wursten 2018), which performs the test for first-order autocorrelation of Born and Breitung (2016). Comparisons with xtistest and xserial (Drukker 2003) are also provided. These tests perform well under stationarity but break down even under mild forms of heteroskedasticity.

Keywords: xtserialpm, heteroskedasticity, fixed-effect model, portmanteau test, serial correlation, short panel data, unbalanced panel.

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1 Introduction

Consider panel data on an outcome $y_{i,t}$ and a set of covariates $x_{i,t}$, where $i = 1, \ldots, N$ and $t = 1, \ldots, T$. The data are independent across groups $i$ but may be dependent within groups. The workhorse specification to analyse such data is the regression model

$$y_{i,t} = x_{i,t}^\top \beta + v_{i,t}, \quad v_{i,t} = \alpha_i + \varepsilon_{i,t},$$

where $\alpha_i$ is a latent individual effect and $\varepsilon_{i,t}$ is an idiosyncratic disturbance whose mean is normalized to zero. These disturbances are taken to be mean independent of the regressors and the individual effects but are otherwise allowed to be (conditionally) heteroskedastic and correlated within each group $i$. Our aim is to test whether the $\varepsilon_{i,t}$ are correlated within groups. Although we do not make it explicit in the notation, everything to follow applies to settings where the panel is unbalanced (possibly with gaps) provided that missingness is at random. As such, the test discussed below is
suitable for data with a group structure; the number of observations on a given group need not be the same and the observations need not be collected over time. Examples are data on student test scores stratified by classroom, or data on individual members of households.

The command \texttt{xtserialpm} that we introduce in this note performs a test for the (multivariate) null of no correlation at any order. The alternative is that at least one error pair is correlated. As such, \texttt{xtserialpm} performs a portmanteau test. The test performed is developed in Jochmans (2019). This test is different from the portmanteau test of Inoue and Solon (2006) that is implemented in \texttt{xtistest} (Wursten 2018) as it is robust to heteroskedasticity. This is important because requiring the errors to have a constant variance within each group is often unrealistic. One situation where heteroskedasticity will arise is when the error process is not in its steady state; this is typical in short panels. A second situation is one where errors are conditionally heteroskedastic and some of the regressors are non-stationary. An example here would be a wage regression where the regressors include such characteristics as age, tenure and experience, and number of children, all of which are non-stationary.

The portmanteau paradigm is to be contrasted with an approach that tests against a specific alternative. Using a portmanteau test is of interest if no strong stand can be taken on the particular form of correlation that should serve as the alternative. This is relevant in many panel data applications, especially when the observations for a given group do not have a natural ordering (such as time, for example). On the other hand, there are cases where attention may be limited to first-order autocorrelation patterns. In such a case, \texttt{xtserial} (Wooldridge 2002, pp. 282–283; Drukker 2003) or its heteroskedasticity-robust version \texttt{xthtest} (Born and Breitung 2016; Wursten 2018) may be of use.\footnote{Under homoskedasticity, \texttt{xtqptest} (Born and Breitung 2016; Wursten 2018) can also be used to test for correlation up to a fixed order.} While tests against specific alternatives may have poor power if the alternative is ill-chosen, they have the advantage that the dimension of the null hypothesis is independent of the sample size. A portmanteau test, on the other hand, necessarily has a null whose dimension grows with the length of the panel, \( T \). Moreover, \texttt{xtserialpm} and \texttt{xtistest} are not well suited for panels where \( T(T - 1)/2 \) is not small relative to \( N \).

The test that is the subject of this paper is introduced in Section 2. The syntax of the Stata command \texttt{xtserialpm} that implements the test is given in Section 3 and
an example is provided in Section 4. The results of a simulation study are given in Section 5. The Monte Carlo analysis compares the performance of \texttt{xtserialpm} with \texttt{xtistest}, \texttt{xtserial}, and \texttt{xthrt} in various settings. While \texttt{xtistest} and \texttt{xtserial} are competitive under homoskedasticity they are unreliable under heteroskedasticity. Although \texttt{xthrt} is designed to be size correct it is found to have very poor power. Moreover, it is virtually unable to detect most violations from the null, even those for which it was designed.

## 2 The test statistic

The presence of the group-level effect $\alpha_i$ complicates the construction of a test based on the data in levels. The approach followed in Jochmans (2019) is to test the null that the difference between all pairwise within-group correlations are zero. There are $T(T-1)/2$ covariances and so

$$q := \frac{T(T-1)}{2} - 1 = \frac{(T+1)(T-2)}{2}$$

linearly-independent differences. There are many ways of selecting $q$ such differences. How they are chosen is irrelevant in practice as each will deliver numerically the same test statistic. A convenient way for notational purposes is as follows. Let $\Delta$ denote the first-differencing operator, i.e., $\Delta \upsilon_{i,t} = \upsilon_{i,t} - \upsilon_{i,t-1}$. Then we test the null

$$H_0 : \mathbb{E}(\upsilon_{i,t'} \Delta \upsilon_{i,t}) = 0$$

for all $t$ and each $t' \leq t - 2$ and $t' = t + 1$.

against the alternative

$$H_1 : \mathbb{E}(\upsilon_{i,t'} \Delta \upsilon_{i,t}) \neq 0$$

for some $t$ and $t' \leq t - 2$ or $t' = t + 1$.

An exercise in adding-up shows that this indeed involves $q$ moments. The rationale for them comes from the observation that

$$\mathbb{E}(\upsilon_{i,t'} \Delta \upsilon_{i,t}) = \mathbb{E}(\varepsilon_{i,t'} \Delta \varepsilon_{i,t}) + \mathbb{E}(\alpha_i \Delta \varepsilon_{i,t})$$

$$= \mathbb{E}(\varepsilon_{i,t'} \Delta \varepsilon_{i,t}),$$

$$= \mathbb{E}(\varepsilon_{i,t'} \varepsilon_{i,t}) - \mathbb{E}(\varepsilon_{i,t'} \varepsilon_{i,t-1})$$

which is indeed the difference between two covariances. The main transition here uses

$$\mathbb{E}(\alpha_i \Delta \varepsilon_{i,t}) = \mathbb{E}(\alpha_i \mathbb{E}(\Delta \varepsilon_{i,t} | \alpha_i)) = 0,$$

which follows from iterated expectations and the assumption that $\mathbb{E}(\varepsilon_{i,t} | \alpha_i) = 0$. 

The $q$ restrictions that make up our null can be written compactly as

$$E(\mathbf{Y}_i^\top D \mathbf{v}_i) = 0,$$

where we have introduced the $(T - 1) \times q$ matrix

$$\mathbf{Y}_i := \begin{pmatrix}
0 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & v_{i,3} & 0 & \cdots & 0 \\
v_{i,1} & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & v_{i,4} & \cdots \\
0 & v_{i,1} & v_{i,2} & 0 & \cdots & 0 & \cdots & 0 & \vdots & \ddots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\
0 & 0 & 0 & 0 & \cdots & v_{i,1} & \cdots & v_{i,T-2} & 0 & 0 & \cdots & 0
\end{pmatrix},$$

and the $T \times 1$ vector $\mathbf{v}_i := (v_{i,1}, \ldots, v_{i,T})^\top$, and write $D$ for the $(T - 1) \times T$ matrix first-difference operator; so $D \mathbf{v}_i = (\Delta v_{i,2}, \ldots, \Delta v_{i,T})^\top$, for example. The left block of the matrix $\mathbf{Y}_i$ is reminiscent of the instrument matrix for GMM estimator of dynamic panel models (see, e.g., Arellano 2003, pp. 88–89). The right block does not appear there as it would not provide valid moment conditions in that context. The null can be tested using a minimum-distance statistic in a sample version of the moment restrictions as soon as three observations per group are available. Note that the dimension of the null grows with $T$. As such, the approach is designed for short panels, where $q$ is small compared to $N$.

To make the test operational the unobserved $\mathbf{v}_{i,t}$ need to be replaced by an estimator. For this an estimator of $\mathbf{\beta}$ is needed. \texttt{xtserialpm} uses the within-group least-squares estimator (as computed by \texttt{xtreg, fe}),

$$\mathbf{b} := \left(\sum_{i=1}^{N} \mathbf{X}_i^\top \mathbf{M} \mathbf{X}_i\right)^{-1} \sum_{i=1}^{N} \mathbf{X}_i^\top \mathbf{M} \mathbf{y}_i,$$

where we have collected all observations for a given group in $\mathbf{y}_i := (y_{i,1}, \ldots, y_{i,T})^\top$ and $\mathbf{X}_i := (\mathbf{x}_{i,1}, \ldots, \mathbf{x}_{i,T})^\top$, and $\mathbf{M}$ denotes the usual $T \times T$ projection matrix that transforms observations into deviations from within-group means. Given $\mathbf{b}$, the residuals

$$u_{i,t} := y_{i,t} - \mathbf{x}_{i,t}^\top \mathbf{b}$$

can be used as estimators of the $\mathbf{v}_{i,t}$.

We then define

$$s_i := \mathbf{U}_i^\top D \mathbf{u}_i - \left(\sum_{j=1}^{N} \mathbf{U}_j^\top D \mathbf{X}_j\right) \left(\sum_{j=1}^{N} \mathbf{X}_j^\top \mathbf{M} \mathbf{X}_j\right)^{-1} \mathbf{X}_i^\top \mathbf{M} \mathbf{u}_i,$$
where the matrix $U_i$ is the sample version of $Y_i$ constructed using the residuals $u_{i,t}$ in place of the unobservable $v_{i,t}$, i.e.,

$$
U_i := \begin{pmatrix}
0 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & u_{i,3} & 0 & \cdots & 0 \\
u_{i,1} & 0 & 0 & 0 & 0 & 0 & u_{i,4} & \vdots \\
0 & u_{i,1} & u_{i,2} & 0 & 0 & 0 & \vdots & \ddots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots & \ddots & \ddots & 0 \\
0 & 0 & 0 & 0 & \cdots & u_{i,1} & \cdots & u_{i,T-2} & 0 & 0 & \cdots & 0
\end{pmatrix},
$$

and we have introduced $u_i := (u_{i,1}, \ldots, u_{i,T})^\top$. The test statistic for our null can then be written as the quadratic form

$$
s^\top V^{-1} s,
$$

where $s := \sum_{i=1}^N s_i$ and $V$ is an estimator of the variance of $s$. 	exttt{xtserialpm} uses the uncentered estimator

$$
V := \sum_{i=1}^N s_i s_i^\top
$$

as default. Use of a centered variance estimator is available as an option. In the simulations reported on below we found that use of the centered estimator is power enhancing but comes at the expense of size distortion in small samples.

Our test statistic has an interpretation that explains its form. Because the second part of $s_i$ sums to zero we have

$$
s = \sum_{i=1}^N s_i = \sum_{i=1}^N U_i^\top D u_i.
$$

This is a sample version of the moments we are aiming to test. The second part of $s_i$ is present to ensure that $V$ is a consistent estimator of the variance of $s$. Moreover, the naive variance estimator $\sum_{i=1}^N (U_i^\top D u_i) (U_i^\top D u_i)^\top$ ignores the fact that the statistic is constructed with residuals rather than (unobservable) errors and will generally not be consistent.

Under the null,

$$
s^\top V^{-1} s \xrightarrow{d} \chi^2_q,
$$

as $N \to \infty$. A test of our null then amounts to comparing the test statistic to the quantiles of the $\chi^2_q$ distribution. Large values are evidence against the null of no serial correlation. This test is consistent against any alternative except the one where all
covariances are constant. Asymptotic power results and calculations for special cases are provided in Jochmans (2019).

3 Stata command xtserialpm

xtserialpm is a stand-alone routine that can run without first running xtreg. The data must be xtset prior to executing xtserialpm. Unbalanced panel data is allowed.

The command has the following syntax:

```
xserialpm depvar [ indepvars ] [ if ] [ in ], [ center, noisily ]
```

The option `center` returns the test statistic computed with centered covariance matrix as discussed above.

The option `noisily` displays the preliminary within-group estimator. The output is the same as that produced by `xtreg, fe`.

Running the command produces a table with the value of the test statistic and the associated $p$-value. The layout of the table mimics the layout of the table produced by xtserial.

The following output is saved to `r`:

- `r(stat)` returns the value of the test statistic;
- `r(df)` returns the degrees of freedom of its limit distribution;
- `r(p)` returns the $p$-value of the test.

Help is available by typing `help xtserialpm`.

4 Example

We use the data from the illustration in Drukker (2003). The following extract loads the data.

```
use http://www.stata-press.com/data/r8/nlswork.dta
(National Longitudinal Survey. Young Women 14-26 years of age in 1968)
```

---

2. This is due to the presence of the fixed effects, and the same is true for all other available tests of serial correlation in a panel setting.
The portmanteau test is computed as

\[ \text{xtserialpm ln_wage c.age##c.age ttl_exp c.tenure##c.tenure i.south if year<=70} \]

and delivers the output

\[ \begin{align*}
\text{Jochmans portmanteau test for within-group correlation in panel data.} \\
\text{H0: no within-group correlation} \\
\text{Chi-sq( 2) } &= 25.658 \\
\text{Prob > Chi-sq } &= 0.0000
\end{align*} \]

The result provides strong evidence for the presence of serial correlation in the errors.

To compute the test statistic using a centered covariance matrix estimator use the `center` option as

\[ \text{xtserialpm ln_wage c.age##c.age ttl_exp c.tenure##c.tenure i.south if year<=70, center} \]

The output is

\[ \begin{align*}
\text{Jochmans portmanteau test for within-group correlation in panel data.} \\
\text{H0: no within-group correlation} \\
\text{Chi-sq( 2) } &= 26.180 \\
\text{Prob > Chi-sq } &= 0.0000
\end{align*} \]

The test statistic is slightly larger and our initial conclusion unaltered.

To perform the test of Inoue and Solon (2006) in this example we first generate residuals from the within group regression by typing

\[ \begin{align*}
\text{quietly xtreg ln_wage c.age##c.age ttl_exp c.tenure##c.tenure i.south if year<=70, fe} \\
predict u, residuals
\end{align*} \]

The test is then performed on these residuals

\[ \text{xtistest u, lags(all)} \]

By default, `xtistest` checks for correlation (in the within-group residuals) up to second
order only. Here, \texttt{xtistest} is invoked with the lag option set to all, so that the command yields the portmanteau test as originally introduced in Inoue and Solon (2006).

The output of the test is

\begin{verbatim}
Inoue and Solon (2006) LM-test on variables u
Panelvar: idcode
Timevar: year

<table>
<thead>
<tr>
<th>Variable</th>
<th>IS-stat</th>
<th>p-value</th>
<th>N</th>
<th>maxT</th>
<th>balance?</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>159.44</td>
<td>0.000</td>
<td>2206</td>
<td>3</td>
<td>gaps</td>
</tr>
</tbody>
</table>
\end{verbatim}

Notes: Under H0, LM \textasciitilde chi2((T-1)(T-2)/2)
H0: No auto-correlation of any order.
Ha: Auto-correlation of some order.

The same conclusion regarding our null is reached.

5 Simulations

We provide size and power comparisons between \texttt{xtserialpm}, \texttt{xtistest}, \texttt{xthrtest}, and \texttt{xtserial}. We consider different specifications for the errors and provide results for different panel dimensions. In all cases, outcomes were generated with fixed effects drawn from a standard normal and with two regressors—the first standard normal and the second zero/one according to the toss of a fair coin—each with a coefficient set to unity. From the time-series literature, we consider alternative specifications where the errors follow an AR(1) or MA(1) process. Both \texttt{xtserial} and \texttt{xthrtest} were designed specifically to detect such forms of serial correlation. It is straightforward to concoct specifications of the error process where these tests will not be able to detect any deviation from the null.

Our first set of results concerns first-order autoregressive error processes of the form

\[ \varepsilon_{i,t} = \rho \varepsilon_{i,t-1} + \eta_{i,t}, \quad t = 2, \ldots, T, \quad |\rho| < 1, \]

where the innovations \( \eta_{i,t} \) are independent standard-normal. Here, the null corresponds

\begin{itemize}
  \item \texttt{xtserialpm} can be applied without modification to the subpanel obtained by dropping all cross sections \( t > T' \). This will deliver a valid test.
  \item If interest lies only in testing that the first \( T' < T \) covariances are the same
\end{itemize}
to $\rho = 0$. In Specification (A1) we draw the initial value $\varepsilon_{i,1}$ from its steady-state distribution. This implies that the error process is strictly stationary (and, hence, homoskedastic). In Specification (A2) we set $\varepsilon_{i,1} = 0$ for all groups. This introduces time-series heteroskedasticity for any value of the autoregressive coefficient. Moreover, we have

$$E(\varepsilon_{i,1}^2) = 0, \quad E(\varepsilon_{i,2}^2) = 1, \quad E(\varepsilon_{i,3}^2) = 1 + \rho^2, \quad E(\varepsilon_{i,4}^2) = 1 + \rho^2 + \rho^4,$$

and so on. The heteroskedasticity is mild but present both under the null and the alternative.

We present simulations results for panels with $N = 100$ and $T \in \{3,6,9\}$. This corresponds to $q \in \{2,14,35\}$ which, relative to $N$, can be considered small, moderate and large. Results are reported in Figure 1 by means of power plots (as obtained over 10,000 replications). For each test the power curve plots the rejection frequency of the test against the value for $\rho \in (-1,1)$ that was used to generate the data. A test is size correct if its rejection frequency under the null equals its size (here set to 5%; the level at which the horizontal axis is set). For a given alternative the rejection frequency is the complement of the probability of making a type II error. Hence, given two tests that are both size correct, the one with a higher rejection frequency is superior. The plots in Figure 1 contain the power curves for the portmanteau tests xtserialpm (full) and xtistest (dashed) as well as for the tests targeted to detect autocorrelation at first-order, xthrtest (dotted) and xtserial (dashed-dotted). Note that xthrtest requires $T \geq 4$ and so is absent from the plots for $T = 3$; $T \geq 3$ suffices for the three other tests.

The left plots shows that, under homoskedasticity, all tests control size well. xtserial performs best here, which is not surprising given that this is the ideal setting for this test. The portmanteau tests both do well and are roughly equally able to reject the null when it is false. xtserialpm does better in the shortest panel while xtistest does better in the longest panel. Both observations arise from the fact that xtserialpm tests more moment conditions. xthrtest lacks power against most alternatives. Although its ability to detect violations from the null improves somewhat with the length of the panel, even in the longest panel considered here it is uniformly outperformed by all other tests.

The right plots shows the impact of time-series heteroskedasticity on all the tests. Being robust to heteroskedasticity, both xtserialpm and xthrtest continue to be size correct. Moreover, the heteroskedasticity improves the power of xtserialpm relative
to the stationary case, especially for $T=3$. \texttt{xttest}, on the other hand, continues to struggle to detect any violation of the null. Both \texttt{xtserial} and \texttt{xttest} are now severely size distorted, with their probability of a type I error far exceeding 5%. Because $|\rho|<1$ the error process is mean reverting and so will become stationary as $t \to \infty$. Moreover, the errors become homoskedastic for large values of $t$. This explains why the performance of \texttt{xtserial} improves as $T$ grows. Of course, no such improvement occurs for \texttt{xttest}. On the other hand, we stress that the properties of \texttt{xtserial} and \texttt{xttest} would not improve if instead $N$ would increase.
Our second set of simulation results involves moving-average processes of order one, that is,
\[ \varepsilon_{i,t} = \eta_{i,t} + \theta \eta_{i,t-1} \quad t = 1, \ldots, T, \quad \theta \in (-\infty, +\infty), \]
where the innovations \( \eta_{i,t} \) are again independent standard-normal. The null corresponds to \( \theta = 0 \). In Specification (B1) we draw the initial value \( \eta_{i,0} \) from the standard normal distribution, again implying stationarity. In Specification (B2) we set \( \eta_{i,0} = 0 \). This leads to heteroskedasticity under the alternative but not under the null. This is different from Specification (A2). Here, as errors are homoskedastic under the null all tests will remain size correct. Note that heteroskedasticity is limited to the first observation, \( \varepsilon_{i,1} \), whose variance is equal to 1; \( \varepsilon_{i,2}, \ldots, \varepsilon_{i,T} \) all have variance \( 1 + \theta^2 \).

Figure 2 provides the power curves for these two specifications. We plot power against the first-order autoregressive coefficient (under stationarity), \( \rho \). This coefficient is one-to-one with \( \theta \) in the sense that
\[ \theta = \frac{1 + \sqrt{1 - 4\rho^2}}{2\rho} \]
when \( \rho \neq 0 \) and \( \theta = 0 \) if \( \rho = 0 \). Note that \( -\frac{1}{2} \leq \rho \leq \frac{1}{2} \). The main conclusions from the autoregressive specifications carry over. Both \text{xtserial} and \text{xtistest} do well under homoskedasticity but have erratic power patterns under heteroskedasticity. \text{xthrttest}, although size correct, continues to be incapable to detect any violation from the null. \text{xtserialpm} performs very well in all specifications and, as such, yields the most reliable test.

6 Conclusion

We have introduced the command \text{xtserialpm} to test for arbitrary patterns of serial correlation in the errors of a fixed-effect regression model estimated from short panel data. Contrary to the existing portmanteau test performed by \text{xtistest} it is robust to heteroskedasticity. Both tests are designed for micropanels. For macropanels, where \( T \) is not small relative to \( N \) only tests against specific alternatives can properly control size. Such tests are implemented in \text{xtserial} and \text{xthrttest}. Simulations evidence shows that even mild forms of heteroskedasticity make the properties of \text{xtistest} and \text{xtserial} break down. Unfortunately, heteroskedasticity in short panels is the rule rather than the exception. Further, \text{xthrttest}, although size-correct under heteroskedasticity, is far less powerful than \text{xtserialpm} even when the alternative under question is characterized
by well-pronounced dependence at first-order. The conclusion from the theory and simulation evidence presented here is that, when heteroskedasticity is suspected, only \texttt{xtserialpm} will provide a suitable test when $T(T - 1)/2$ is small compared to $N$. On the other hand, only \texttt{xthrtest} is guaranteed to be size-correct when $T(T - 1)/2$ is large relative to $N$. However, it may not pick up violations from the null if they do not occur at first-order. Furthermore, even if they do, our simulations show that the test needs the sample size to be substantial to be able to safeguard against type II errors with reasonable probability.
7 References


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